Processing Compressed Texts: A Tractability Border

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Combinatorial Pattern Matching 2007

Consider two compressed texts

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$=?$

Can we say whether they are equal without unpacking them?

Outline

1. Popular topic in stringology: **algorithms for compressed texts**

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2. New algorithm for fully compressed pattern matching
3. Tractability border for processing compressed texts
4. Open problems

Part I

Formulating the problem

Straight-line Programs: Definition

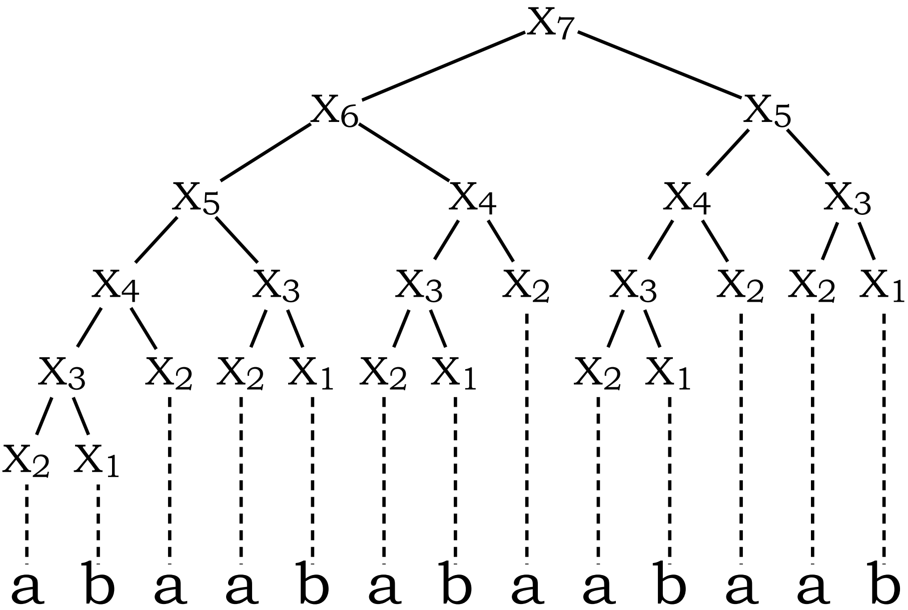
Straight-line program (SLP) is a context-free grammar generating **exactly one** string

Two types of productions: $X_i \longrightarrow a$ and $X_i \longrightarrow X_p X_q$

Example

abaababaabaab

$$
\begin{aligned}
X_1 &\longrightarrow b \\
X_2 &\longrightarrow a \\
X_3 &\longrightarrow X_2 X_1 \\
X_4 &\longrightarrow X_3 X_2 \\
X_5 &\longrightarrow X_4 X_3 \\
X_6 &\longrightarrow X_5 X_4 \\
X_7 &\longrightarrow X_6 X_5
\end{aligned}
$$


X_7
X_6 X_5
X_5 X_4 X_4 X_3
X_4 X_3 X_3 X_2 X_3 X_2 X_2 X_1
X_3 X_2 X_2 X_1 X_2 X_1 X_2 X_1
X_2 X_1
a b a a b a b a a b a a b

Rytter Theorem

Theorem (Rytter'03)

Resulting archive of most practical compression methods can be transformed into SLP generating the same original text. Two nice properties:

1. *SLP has almost the same size as initial archive*
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Notable exception: compression schemes based on Burrows-Wheeler transform

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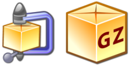
- **Connections to practical problems:**
  software verification, database compression, multimedia search
- **Complexity surprises:**
  many problems have unexpected computational complexity when input is given in compressed form

Comparison Problems



Consider two straight-line programs $\mathcal{P}, \mathcal{Q}$ generating texts P and Q. Tasks:

1. Determine whether $P = Q$
2. **Fully compressed pattern matching:** check whether P is a substring of Q
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Let m, n be the sizes of $\mathcal{P}, \mathcal{Q}$. Best previously known result is $\mathcal{O}(n^2 m^2)$ algorithm for (1) and (2) **MST'97**

Example

Do they generate the same text?

$$
\begin{array}{ll}
X_1 \longrightarrow b & Y_1 \longrightarrow b \\
X_2 \longrightarrow a & Y_2 \longrightarrow a \\
X_3 \longrightarrow X_2X_1 & Y_3 \longrightarrow Y_1Y_2 \\
X_4 \longrightarrow X_3X_2 & Y_4 \longrightarrow Y_2Y_3 \\
X_5 \longrightarrow X_4X_3 & Y_5 \longrightarrow Y_3Y_4 \\
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X_7 \longrightarrow X_6 X_5 & Y_7 \longrightarrow Y_5 Y_6
\end{array}
$$

Part II

Fully compressed pattern matching

New FCPM Algorithm

MAIN RESULT 1

Fully compressed pattern matching can be solved in $\mathcal{O}(n^2 m)$ time

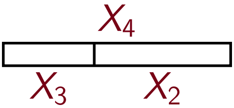
Dynamic Programming (1/2)

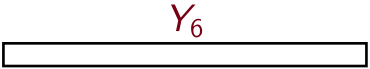
$X_1 \rightarrow b$
$X_2 \rightarrow a$
$X_3 \rightarrow X_2 X_1$
$X_4 \rightarrow X_3 X_2$
$X_5 \rightarrow X_4 X_3$
$X_6 \rightarrow X_5 X_4$

$Y_1 \rightarrow b$
$Y_2 \rightarrow a$
$Y_3 \rightarrow Y_1 Y_2$
$Y_4 \rightarrow Y_2 Y_3$
$Y_5 \rightarrow Y_3 Y_4$
$Y_6 \rightarrow Y_4 Y_5$
$Y_7 \rightarrow Y_5 Y_6$

For every i, j we compute occurrences of X_i in Y_j in lexicographic order of pairs: from $(1, 1)$ to $(6, 7)$

Dynamic Programming (2/2)

Searching 

in 

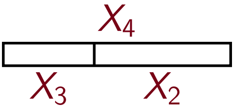
From previous steps of dynamic programming:

X_3 occurrences in Y_6

X_2 occurrences in Y_6.

Dynamic Programming (2/2)

Searching



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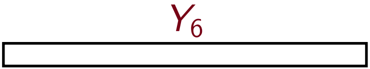

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Take X_3 occurrences, X_2 occurrences, shift and intersect!

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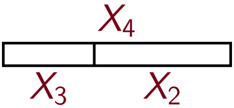
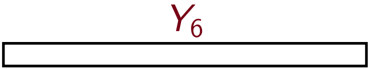
Searching 

in 

From previous steps of dynamic programming:

- X_3 occurrences in Y_6
- X_2 occurrences in Y_6.

Take X_3 occurrences, X_2 occurrences, shift and intersect!

Complexity of one step of dynamic programming: $\mathcal{O}(n)$
Number of steps in dynamic programming: $\mathcal{O}(nm)$

Part III

Tractability Border

Check Your Intuition

When two SLPs of sizes n, m are given we can check equivalence between texts generated by them in time $\mathcal{O}(\min(n^2 m, m^2 n))$

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What is the complexity of computing Hamming distance between two SLP-generated texts?

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What is the complexity of computing Hamming distance between two SLP-generated texts?

MAIN RESULT 2

The problem of computing Hamming distance between SLP-generated texts is #P-complete

Tractability Border

When input texts are compressed:

∃ poly algorithm:

GKPR'96 Equivalence
GKPR'96 Regular Language Membership
KPR'95 Fully Compressed Pattern Matching
CGLM'06 Window Subsequence Matching
GKPR'96 Shortest Period
L'07 Shortest Cover

At least NP-hard:

L'07 Hamming distance
Lohrey'04 Context-Free Language Membership
LL'06 Fully Compressed Subsequence Matching
BKLPR'02 Two-dimensional Compressed Pattern Matching

Part IV
Open Problems

OP1: Longest Common Substring

Input: Two SLPs generating texts P and Q

Task: Compute the length of the longest common substring of P and Q

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Can we do it in polynomial time?

OP2: SLP for OR

Input: Two SLPs of sizes n, m generating binary strings P and Q of the same length

Task: Compute a close-to-minimal SLP generating "bitwise OR between P and Q"

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Input: Two SLPs of sizes n, m generating binary strings P and Q of the same length

Task: Compute a close-to-minimal SLP generating "bitwise OR between P and Q"

Can we do it in time $poly(n + m + output)$?

OP3: Edit Distance

Input: Text P of length l and SLP of size m generating text Q

Input: Compute edit distance between P and Q

OP3: Edit Distance

Input: Text P of length l and SLP of size m generating text Q

Input: Compute edit distance between P and Q

Can we do it in $\mathcal{O}(lm)$ time?

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- Straight-line programs is a wide-accepted model of compressed text
- Fully compressed pattern matching can be solved in $\mathcal{O}(n^2 m)$ time
- Computing Hamming distance between compressed texts is #P-complete
- Open problems: longest common substring, bitwise OR and edit distance for compressed texts

Last Slide

Search "**Lifshits**" or visit http://logic.pdmi.ras.ru/~yura/

- Yury Lifshits

  Processing Compressed Texts: A Tractability Border

  CPM'07 // on-line version

- Yury Lifshits and Markus Lohrey

  Querying and Embedding Compressed Texts

  MFCS'06 // on-line version

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Thank you for your attention! Questions?